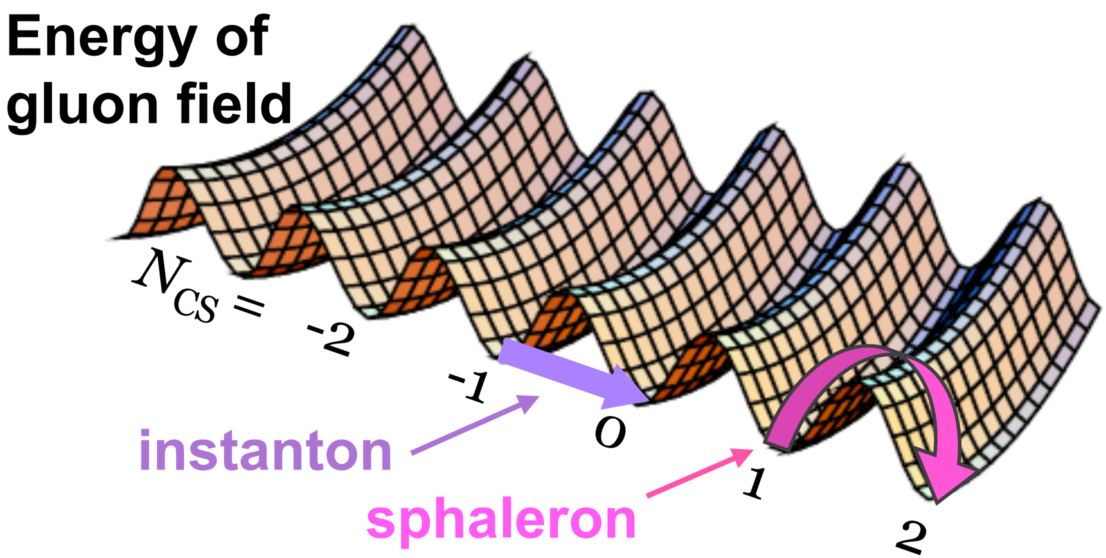
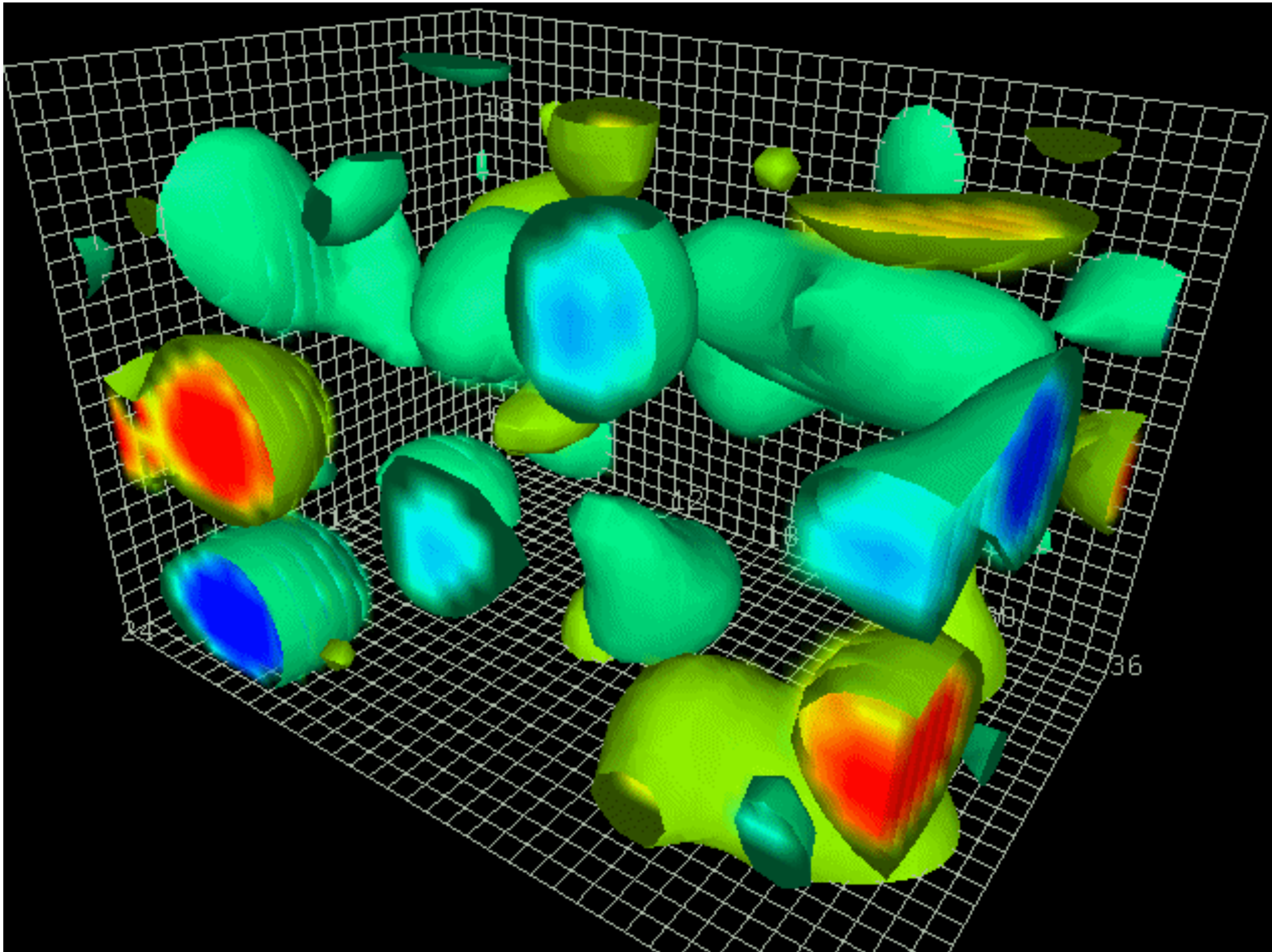
Topology and QCD vacuum

The instanton solutions in Minkowski space-time describe the tunneling events between the topological sectors of the vacuum marked by different integer values of $N_{CS} \equiv \int d^3x K_o$

$$K_\mu = \frac{1}{16\pi^2}\epsilon_{\mu\alpha\beta\gamma}\left(A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3} f^{abc} A^a_\alpha A^b_\beta A^c_\gamma\right)$$



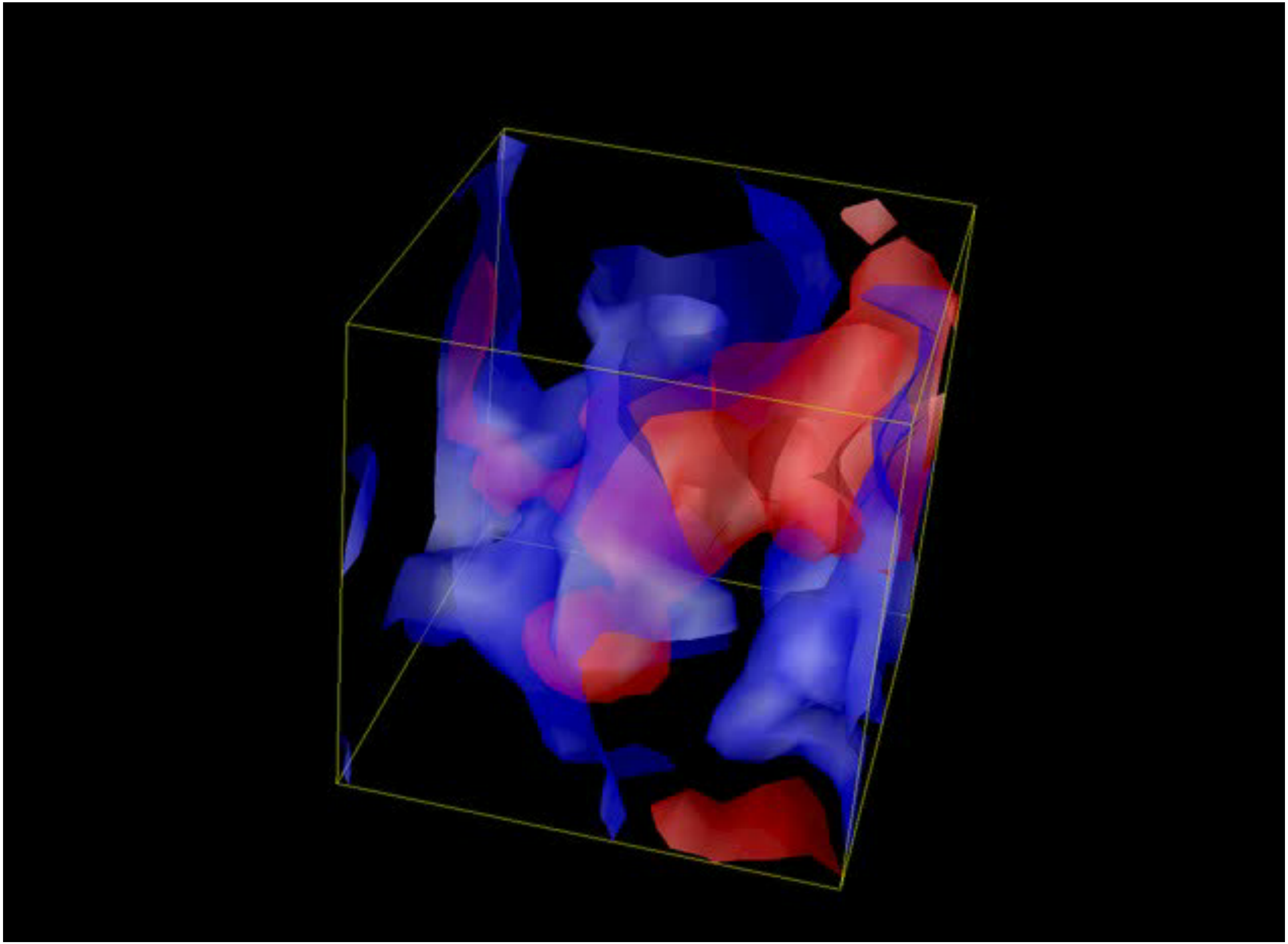
Topological number fluctuations in QCD vacuum
("cooled" configurations)



D. Leinweber

Topological number fluctuations in QCD vacuum

ITEP Lattice Group

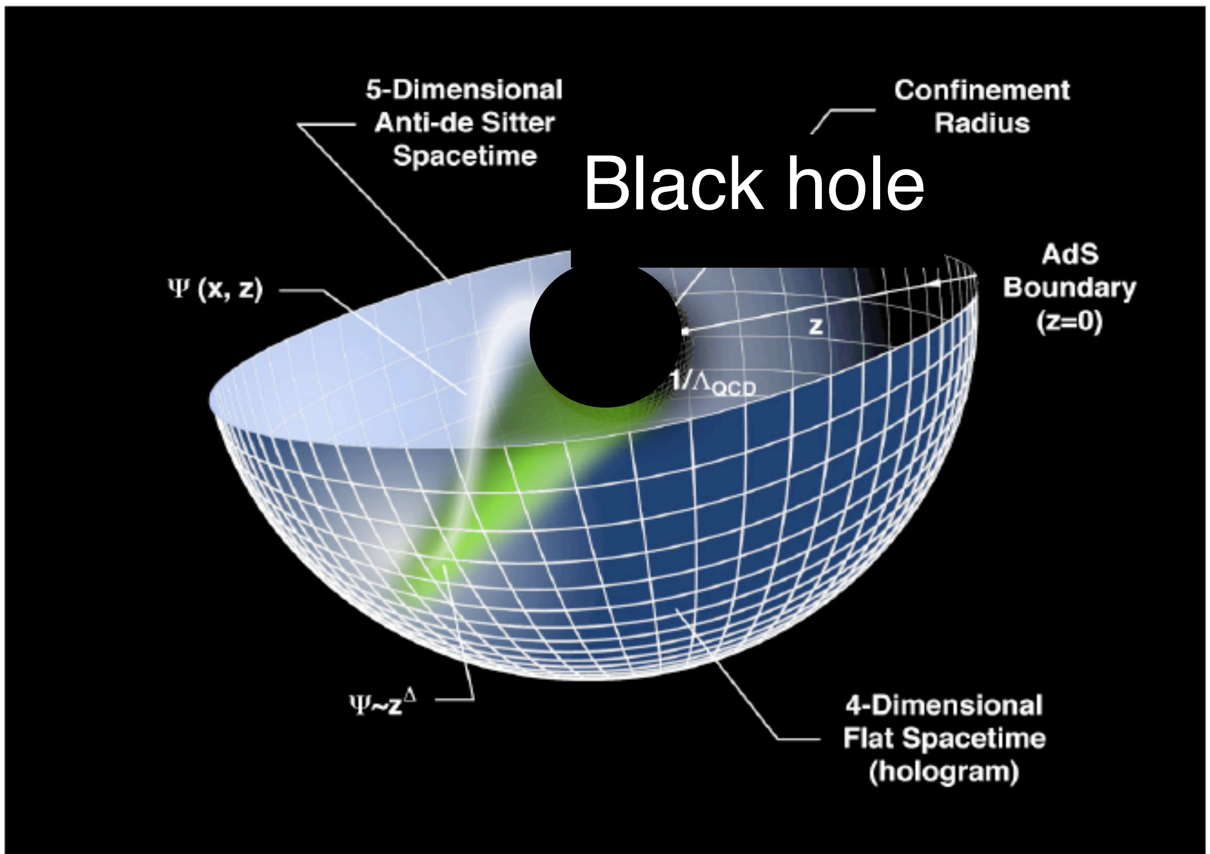


Topological number diffusion at strong coupling

Chern-Simons number
diffusion rate
at strong coupling

$$\Gamma = \frac{(g_{\rm YM}^2 N)^2}{256\pi^3} T^4$$

D.Son,
A.Starinets
hep-th/
020505



NB: This calculation is analogous to the calculation of shear viscosity that led to the “perfect liquid”

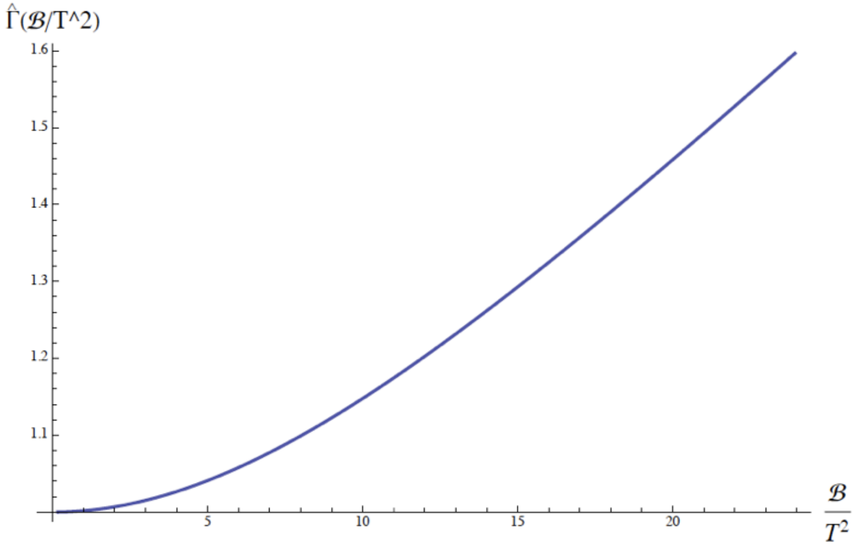
The Chern-Simons diffusion rate in an external magnetic field

strongly coupled N=4 SYM plasma in an external $U(1)_R$ magnetic field through holography

G. Basar, DK, Phys Rev D, arXiv:1202.2161

weak field:

$$\Gamma_{CS} = \frac{(g^2 N)^2}{256\pi^3} T^4 \left(1 + \frac{1}{6\pi^4}\frac{\mathcal{B}^2}{T^4} + \mathcal{O}(\frac{\mathcal{B}^4}{T^8})\right)$$



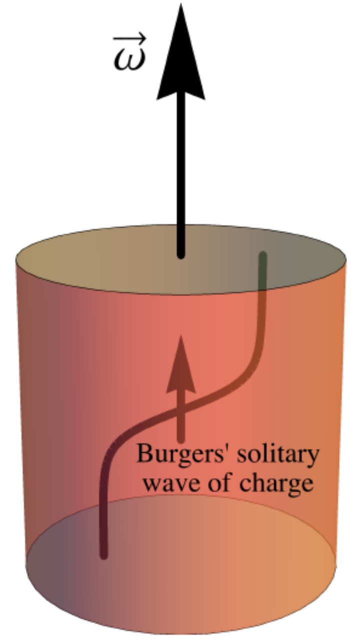
strong field increases the rate:

$$\Gamma(\mathcal{B}, T) = \frac{(g^2 N)^2}{384\sqrt{3}\pi^5} \mathcal{B}\, T^2$$

dimensional reduction

G.Basar, DK, H.-U.Yee, PhysRevB89(2014)035142

Anomalous transport induced by vorticity



Consider a "hot" system (QGP, DSM) with $\frac{\mu}{T} \ll 1$

The chemical potential is then proportional to charge density:

$$\mu \approx \chi^{-1}\rho + \mathcal{O}\left(\rho^3\right)$$

the CME current is

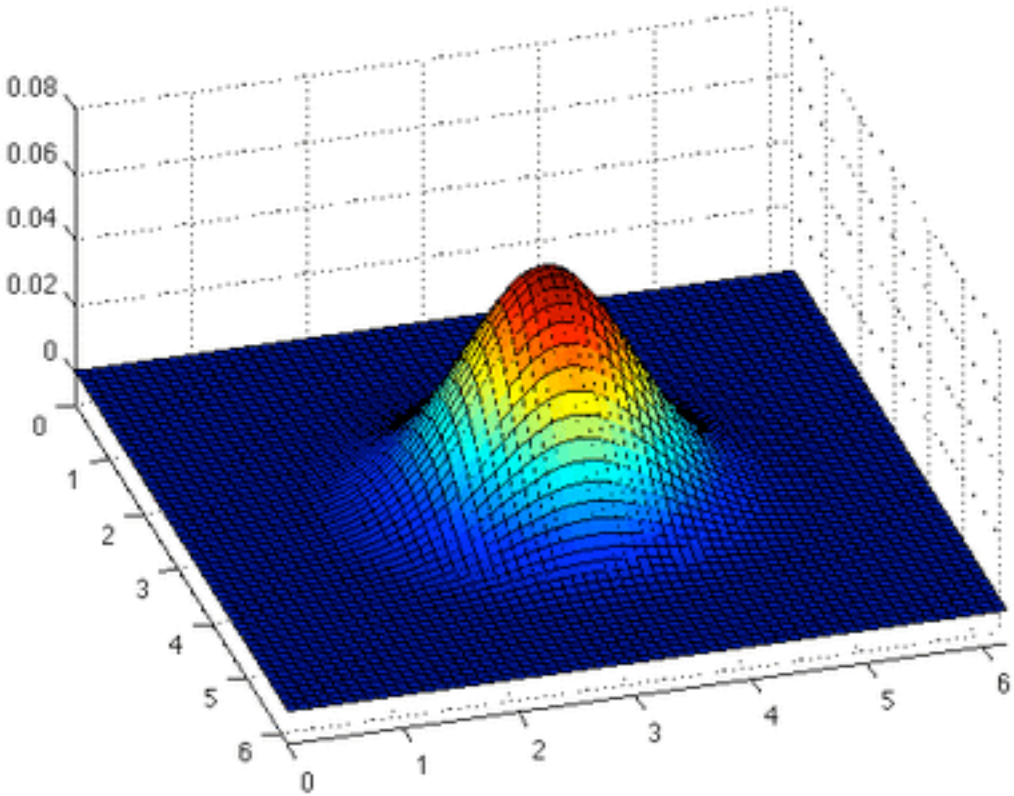
$$J^3 = \frac{ke}{4\pi^2}\left(\chi^{-2}\rho^2 + \frac{\pi^2}{3}T^2\right)\omega - D\partial_3\rho + \mathcal{O}\left(\partial^2, \rho^3\right)$$

and the charge conservation $\partial_t\rho + \partial_3 J^3 = 0$ leads to

$$\partial_t\rho + C\rho\partial_x\rho - D\partial_x^2\rho = 0 \qquad C = \frac{ke\omega}{2\pi^2\chi^2} \qquad x \equiv x^3$$

The Burgers’ equation

$$\partial_t \rho + C \rho \partial_x \rho - D \partial_x^2 \rho = 0$$



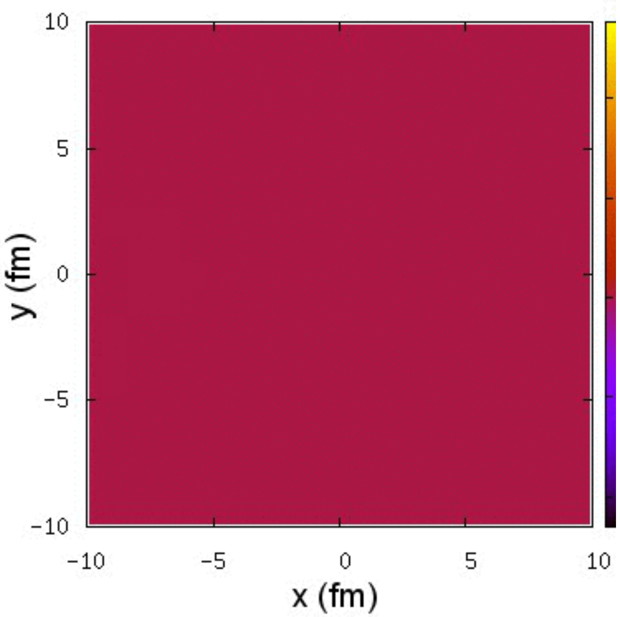
Exactly soluble by Cole-Hopf transformation -

initial value problem, integrable dynamics

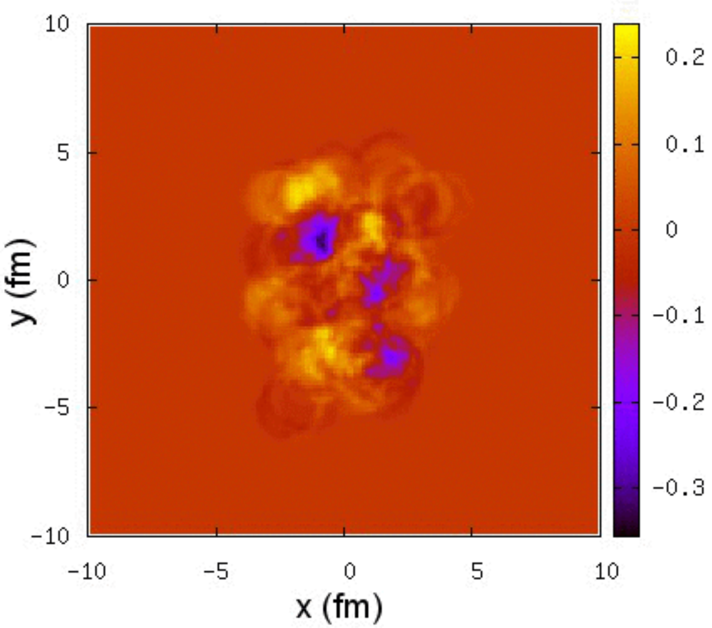
describes shock waves, solitons, ...

CMHD

Electric charge



Chiral charge



Y.Hirono, T.Hirano, DK, (Stony Brook – Tokyo), arxiv:1412.0311
(3+1) ideal CMHD (Chiral MagnetoHydroDynamics)

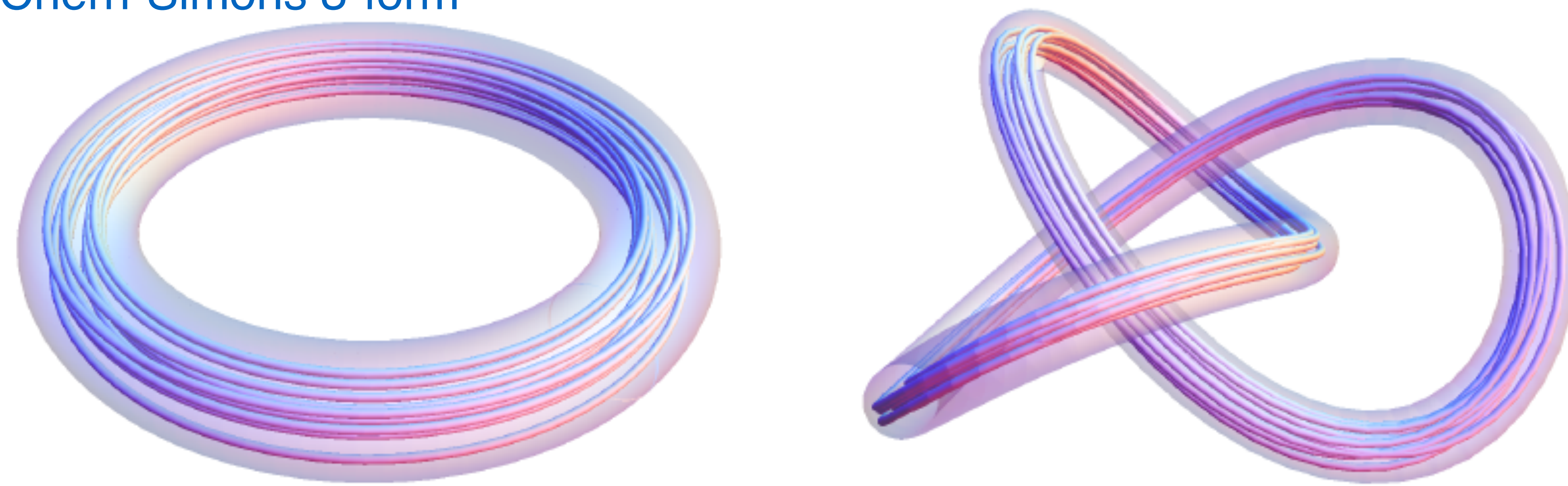
BEST Theory Collaboration (DOE)

Quantized CME from knot reconnections

Y. Hirono, DK, Y. Yin, PRL 117(2016) 172301

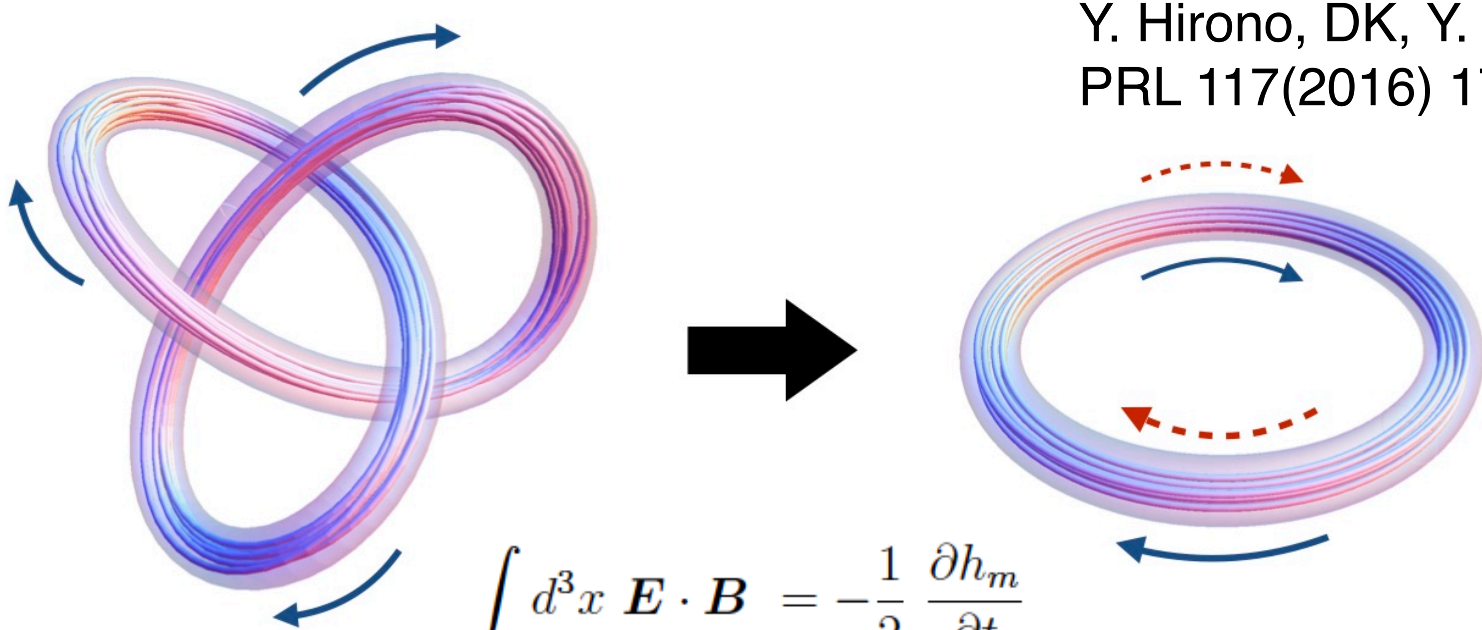
Magnetic helicity is the measure of “knottedness” of magnetic flux - Chern-Simons 3-form

$$h_m \equiv \int d^3x \, \boldsymbol{A} \cdot \boldsymbol{B}$$



Consider a tube (unknot) of magnetic flux, with chiral fermions localized on it.
To turn it into a (chiral) knot, we need a magnetic reconnection.
What happens to the fermions during the reconnection?

Y. Hirono, DK, Y. Yin,
PRL 117(2016) 172301



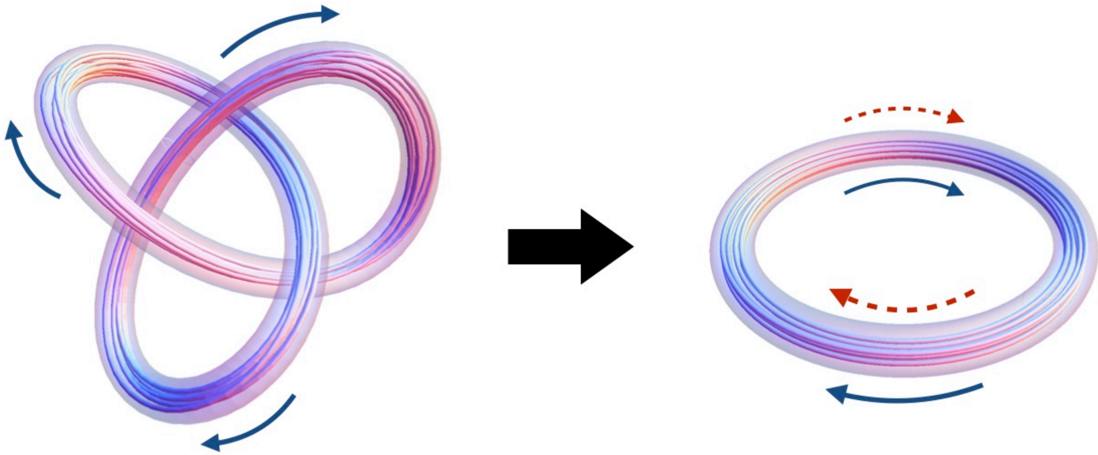
$$\int d^3x \, \boldsymbol{E} \cdot \boldsymbol{B} = -\frac{1}{2} \frac{\partial h_m}{\partial t}$$

Changing magnetic flux through the area spanned by the tube will generate the electric field (Faraday's induction):

$$\frac{d}{dt}\Phi_B = -\oint_C \boldsymbol{E} \cdot d\boldsymbol{x}$$

The electric field will generate electric current of fermions (chiral anomaly in 1+1 D):

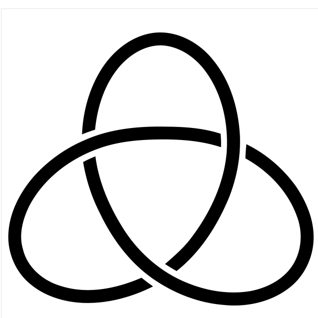
$$\Delta J = \Delta J_R + \Delta J_L = \frac{q^3 \Phi^2}{2\pi^2 L}$$



Y. Hirono, DK, Y. Yin,
PRL 117(2016) 172301

Helicity change per magnetic reconnection is $\Delta\mathcal{H} = 2\Phi^2.$

Multiple magnetic reconnections leading to non-chiral knots do not induce net current (need to break left-right symmetry).



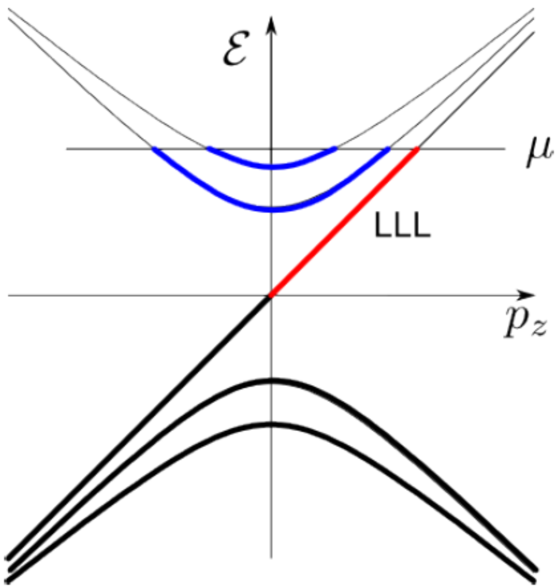
For N_+ positive and N_- negative crossings on a planar knot diagram, the total magnetic helicity is:

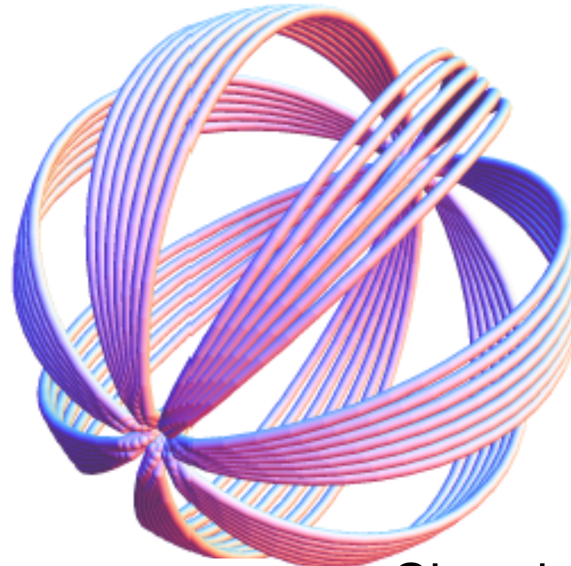
$$\mathcal{H} = 2(N_+ - N_-)\Phi^2$$

The total current induced by reconnections to a chiral knot:

$$\boxed{J = \frac{q^3\mathcal{H}}{4\pi^2 L}}$$

Chirality transfer from fermions to magnetic helicity





Chandrasekhar-Kendall states (ApJ, 1957)

$$\partial_\mu j^\mu_A = C_{\rm A} \boldsymbol{E}\cdot\boldsymbol{B}$$

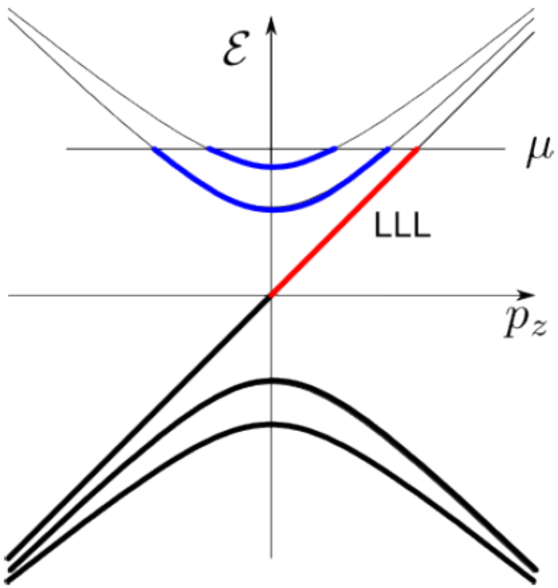
$$h_m \equiv \int d^3x\ \boldsymbol{A}\cdot\boldsymbol{B}$$

$$\int d^3x\ \boldsymbol{E}\cdot\boldsymbol{B} = -\frac{1}{2}\frac{\partial h_m}{\partial t}$$

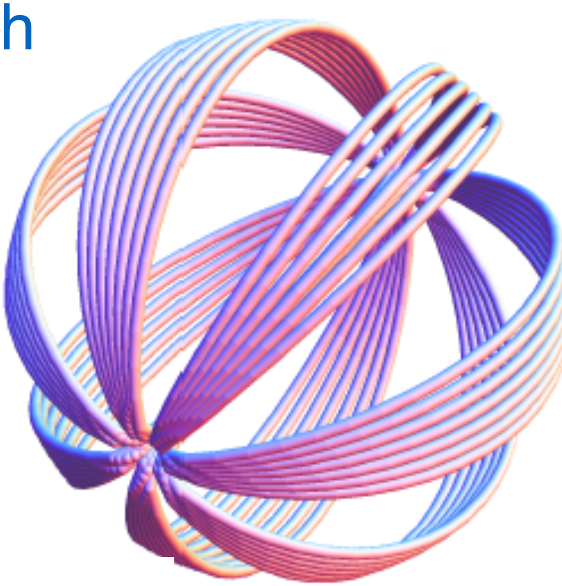
$$h_0 \equiv h_m + h_F = \text{const}$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031

Inverse cascade of magnetic helicity



Instability at k< $c_A \, \mu_A$ leads to the growth of magnetic helicity

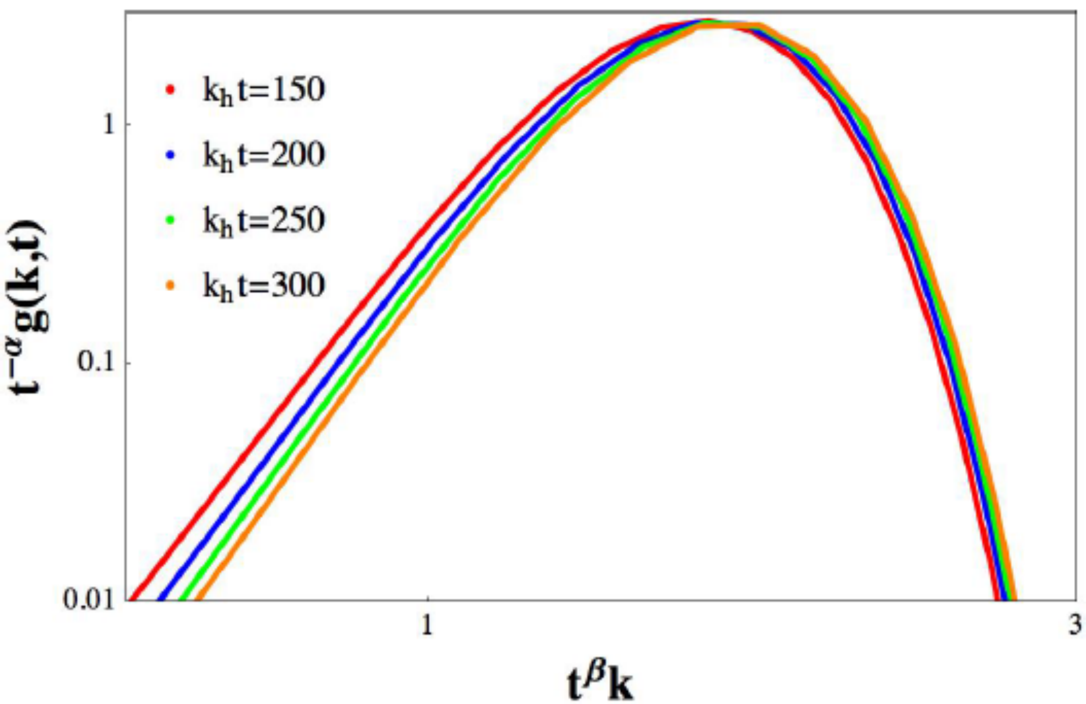


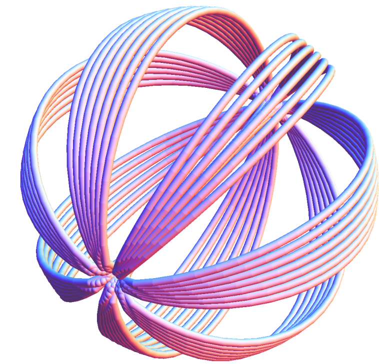
Increase of of magnetic helicity reduces μ_A

Inverse cascade:

M.Joyce and M.Shaposhnikov, PRL 79 (1997) 1193;
R.Jackiw and S.Pi, PRD 61 (2000) 105015;
A.Boyarsky, J.Frohlich, O.Ruchayskiy, PRL 108 (2012) 031301;
PRD 92 (2015) 043004;
H.Tashiro, T.Vachaspati, A.Vilenkin, PRD 86 (2012) 105033

Self-similar cascade of magnetic helicity driven by CME





$$g(k,t) \sim t^{\alpha}\tilde{g}(t^{\beta}k) \qquad \alpha = 1\,, \qquad \beta = 1/2$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031;
N. Yamamoto, Phys.Rev.D93 (2016) 125016