Topology and QCD vacuum

The instanton solutions in Minkowski space-time describe the tunneling events between the topological sectors of the vacuum marked by different integer values of

$$N_{CS} \equiv \int d^3 x K_o$$

$$K_\mu = \frac{1}{16\pi^2} \epsilon_{\mu\alpha\beta\gamma} \left( A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3} f^{abc} A^a_\alpha A^b_\beta A^c_\gamma \right)$$

Energy of gluon field

\(N_{CS} = -2, -1, 0, 1, 2\)

instanton

sphaleron
Topological number fluctuations in QCD vacuum
(“cooled” configurations)
Topological number fluctuations in QCD vacuum
ITEP Lattice Group
Topological number diffusion at strong coupling

Chern-Simons number diffusion rate at strong coupling

\[ \Gamma = \frac{(g_{YM}^2 N)^2}{256\pi^3} T^4 \]

D. Son, A. Starinets
hep-th/020505

NB: This calculation is analogous to the calculation of shear viscosity that led to the “perfect liquid”
The Chern-Simons diffusion rate in an external magnetic field

strongly coupled N=4 SYM plasma in an external U(1)$_{R}$ magnetic field through holography


weak field:

$$\Gamma_{CS} = \frac{(g^2 N)^2}{256\pi^3} T^4 \left(1 + \frac{1}{6\pi^4} \frac{B^2}{T^4} + O\left(\frac{B^4}{T^8}\right)\right)$$

strong field increases the rate:

$$\Gamma(B, T) = \frac{(g^2 N)^2}{384\sqrt{3}\pi^5} B T^2$$

dimensional reduction
Anomalous transport induced by vorticity

Consider a “hot” system (QGP, DSM) with $\frac{\mu}{T} \ll 1$

The chemical potential is then proportional to charge density:

$$\mu \approx \chi^{-1} \rho + \mathcal{O} \left( \rho^3 \right)$$

the CME current is

$$J^3 = \frac{k e}{4 \pi^2} \left( \chi^{-2} \rho^2 + \frac{\pi^2}{3} T^2 \right) \omega - D \partial_3 \rho + \mathcal{O} \left( \partial^2, \rho^3 \right)$$

and the charge conservation

$$\partial_t \rho + \partial_3 J^3 = 0$$

leads to

$$\partial_t \rho + C \rho \partial_x \rho - D \partial_x^2 \rho = 0$$

$$C = \frac{k e \omega}{2 \pi^2 \chi^2} \quad x \equiv x^3$$

G. Basar, DK, H.-U. Yee,
PhysRevB89(2014)035142
The Burgers’ equation

\[ \partial_t \rho + C \rho \partial_x \rho - D \partial_x^2 \rho = 0 \]

Exactly soluble by Cole-Hopf transformation -

initial value problem, integrable dynamics

describes shock waves, solitons, ...
Y. Hirono, T. Hirano, DK, (Stony Brook – Tokyo), arxiv:1412.0311
(3+1) ideal CMHD (Chiral MagnetoHydroDynamics)

BEST Theory Collaboration (DOE)
Quantized CME from knot reconnections

Magnetic helicity is the measure of “knottedness” of magnetic flux
- Chern-Simons 3-form

Consider a tube (unknot) of magnetic flux, with chiral fermions localized on it.
To turn it into a (chiral) knot, we need a magnetic reconnection. What happens to the fermions during the reconnection?

Y. Hirono, DK, Y. Yin, PRL 117(2016) 172301
Changing magnetic flux through the area spanned by the tube will generate the electric field (Faraday’s induction):

\[ \int d^3x \, E \cdot B = -\frac{1}{2} \frac{\partial h_m}{\partial t} \]

The electric field will generate electric current of fermions (chiral anomaly in 1+1 D):

\[ \frac{d}{dt} \Phi_B = - \oint_C E \cdot d\mathbf{x} \]

\[ \Delta J = \Delta J_R + \Delta J_L = \frac{q^3 \Phi^2}{2\pi^2 L} \]
Helicity change per magnetic reconnection is 
$$\Delta \mathcal{H} = 2\Phi^2.$$ 

Multiple magnetic reconnections leading to non-chiral knots do not induce net current (need to break left-right symmetry).

For $N_+$ positive and $N_-$ negative crossings on a planar knot diagram, the total magnetic helicity is:

$$\mathcal{H} = 2(N_+ - N_-)\Phi^2$$

The total current induced by reconnections to a chiral knot:

$$J = \frac{q^3 \mathcal{H}}{4\pi^2 L}$$

Y. Hirono, DK, Y. Yin, PRL 117(2016) 172301
Chirality transfer from fermions to magnetic helicity

\[ h_m \equiv \int d^3x \ A \cdot B \]
\[ h_0 \equiv h_m + h_F = \text{const} \]
\[ \partial_\mu j_\mu^A = C_A E \cdot B \]
\[ \int d^3x \ E \cdot B = -\frac{1}{2} \frac{\partial h_m}{\partial t} \]


Inverse cascade of magnetic helicity

Instability at $k < C_A \mu_A$ leads to the growth of magnetic helicity

Increase of magnetic helicity reduces $\mu_A$

Inverse cascade:
M.Joyce and M.Shaposhnikov, PRL 79 (1997) 1193;
R.Jackiw and S.Pi, PRD 61 (2000) 105015;
A.Boyarsky, J.Frohlich, O.Ruchayskiy, PRL 108 (2012) 031301;
PRD 92 (2015) 043004;
Self-similar cascade of magnetic helicity driven by CME

\[ g(k, t) \sim t^{\alpha} \tilde{g}(t^{\beta} k) \quad \alpha = 1, \quad \beta = 1/2 \]