Hydrodynamics and symmetries

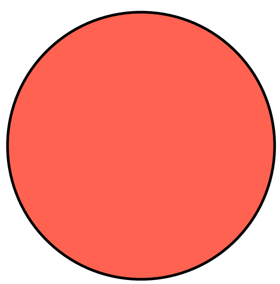
- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

Son, Surowka; Landsteiner, Megias, Pena-Benitez; Sadofyev, Isachenkov; Kalaydzhyan, Kirsch; DK, Yee; Zakharov; Jensen, Loganayagam, Yarom; Neiman, Oz; ….

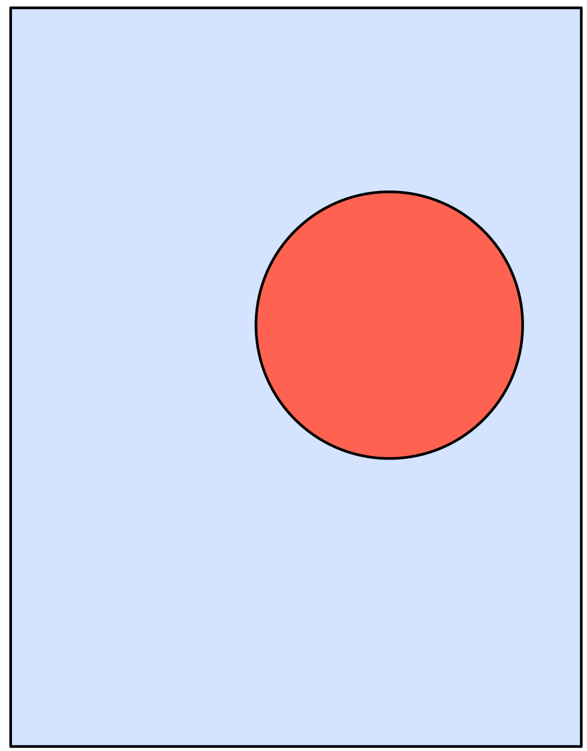
No entropy production from P-odd anomalous terms

DK and H.-U. Yee, 1105.6360; PRD

Entropy grows



$$\partial_\mu s^\mu \geq 0$$



Mirror reflection: entropy decreases ?

$$\partial_\mu s^\mu \leq 0$$

Decrease is ruled out by 2nd law of thermodynamics

$$\partial_\mu s^\mu = 0$$

Allows to compute analytically 13 out of 18 anomalous transport coefficients in 2nd order relativistic hydrodynamics

DK and H.-U. Yee, 1105.6360; PRD

Conformally invariant Chiral magnetohydrodynamics

$$T^{\mu\nu\cdots}_{\alpha\beta\cdots}(x) \to e^{w\phi(x)} T^{\mu\nu\cdots}_{\alpha\beta\cdots}(x)$$

$$w = [\text{mass dimension}] + [\#\text{ of upper indices}] - [\#\text{ of lower indices}]$$

$$\mathcal{D}_\mu f = \nabla_\mu f + w\mathcal{W}_\mu$$

$$\mathcal{W}_\mu = u^\nu \nabla_\nu u_\mu - \frac{(\nabla_\nu u^\nu)}{3} u_\mu$$

$$\begin{aligned}
&\sigma^{\mu\nu}\mathcal{D}_\nu\bar{\mu}\ ,\ \omega^{\mu\nu}\mathcal{D}_\nu\bar{\mu}\ ,\ \Delta^{\mu\nu}\mathcal{D}^\alpha\sigma_{\nu\alpha}\ ,\ \Delta^{\mu\nu}\mathcal{D}^\alpha\omega_{\nu\alpha}\ ,\ \sigma^{\mu\nu}\omega_\nu\ ,\\
&\sigma^{\mu\nu}E_\nu\ ,\ \sigma^{\mu\nu}B_\nu\ ,\ \omega^{\mu\nu}E_\nu\ ,\ \omega^{\mu\nu}B_\nu\ , u^\nu\mathcal{D}_\nu E^\mu\ , \qquad (2.60)\\
&\epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha\mathcal{D}_\beta\bar{\mu}\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu B_\alpha\mathcal{D}_\beta\bar{\mu}\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha B_\beta\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha E_\beta\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha B_\beta\ .
\end{aligned}$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\omega_{\alpha\beta} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\nabla_\alpha u_\beta \qquad E^\mu = F^{\mu\nu}u_\nu \quad , \quad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

$$\sigma_{\mu\nu} = \frac{1}{2}\left(\mathcal{D}_\mu u_\nu + \mathcal{D}_\nu u_\mu\right) \quad , \quad \omega_{\mu\nu} = \frac{1}{2}\left(\mathcal{D}_\mu u_\nu - \mathcal{D}_\nu u_\mu\right) \quad , \quad \mathcal{D}_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu}$$

Problem:

Derive the equations of Chiral Magnetohydrodynamics
for non-conformal fluids assuming that the breaking
of conformal invariance is due to the scale anomaly,

$$\partial_\mu s^\mu = \theta^\mu_\mu$$

$$s^\mu = x_\nu \; \theta^{\mu\nu}$$

In some SUSY theories with electric-magnetic duality,
scale and chiral anomaly are related,
so this should reflect on hydrodynamics.

Why is this relevant?

Scale invariance

Scale transformations (dilatations) are defined by

$$x \to e^{\lambda}\ x$$

the corresponding dilatational current is

$$s^{\mu} = x_{\nu}\ \theta^{\mu\nu}$$

Hermann Weyl
(1885-1955)

It is conserved
(a theory is scale-invariant)
if the energy-momentum is traceless:

$$\partial_{\mu} s^{\mu} = \theta^{\mu}_{\mu}$$

Scale invariance in QCD

The trace of the energy-momentum tensor in QCD (computed in classical field theory) is

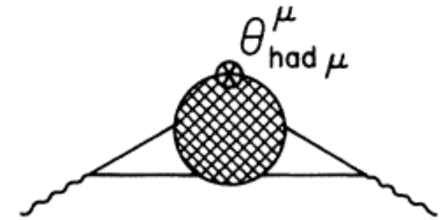
$$\Theta^{\alpha}_{\alpha} = \sum_{l=u,d,s} m_l \; \bar{q}_l q_l + \sum_{h=c,b,t} m_h \; \bar{q}_h q_h$$

Two problems:

1. Potentially large contribution from heavy quarks to the masses of light hadrons

2. If we forget about heavy quarks, all hadron masses must be equal to zero in the chiral limit

Scale anomaly in QCD



The quantum effects (loop diagrams) modify the expression for the trace of the energy-momentum tensor:

$$\Theta^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1+\gamma_{m_h}) \bar{Q}_h Q_h,$$

Running coupling -> dimensional transmutation -> mass scale

Gross, Wilczek; Politzer

$$\beta(g) = -b \frac{g^3}{16\pi^2} + \ldots, \quad b = 9 - \frac{2}{3} n_h,$$

Ellis, Chanowitz; Crewther; Collins, Duncan, Joglecar; …

At small momentum transfer, heavy quarks decouple:

$$\sum_h m_h \bar{Q}_h Q_h \to -\frac{2}{3}\, n_h\, \frac{g^2}{32\pi^2} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \ldots$$

SVZ ’78

so only light quarks enter the final expression

$$\Theta^{\alpha}_{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \sum_{l=u,d,s} m_l \bar{q}_l q_l,$$

The proton mass

At zero momentum transfer, the matrix elements of the energy-momentum tensor are

$$\langle P|\theta^{\mu\nu}|P\rangle = 2P^{\mu}P^{\nu}$$

so that the trace of the energy-momentum tensor defines the masses of hadrons:

$$\langle P|\theta^{\mu}_{\mu}|P\rangle = 2M^2$$

$$\Theta^{\alpha}_{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \sum_{l=u,d,s} m_l \bar{q}_l q_l,$$

In the chiral limit, the entire mass is from gluons!

The proton mass

At finite quark mass, contribution from "sigma-terms"

$$\Sigma_{\pi N} = \hat{m}\langle p|\bar{u}u + \bar{d}d|p\rangle$$

can be extracted from pion-nucleon scattering or measured on the lattice

e.g. Y.-B.Yang et al arXiv:1511.09089

Sometimes interpreted as either

1. Contribution from quark masses

or

1. Contribution from chiral symmetry breaking

But the interpretation is more subtle

The proton mass

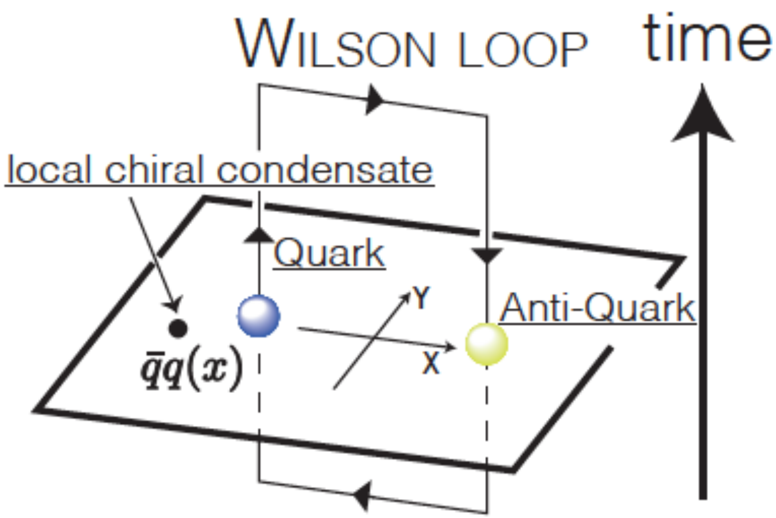
The matrix elements over a hadron state
have to be understood as the **difference** of the value of the measured quantity in the hadron and in the vacuum, e.g.

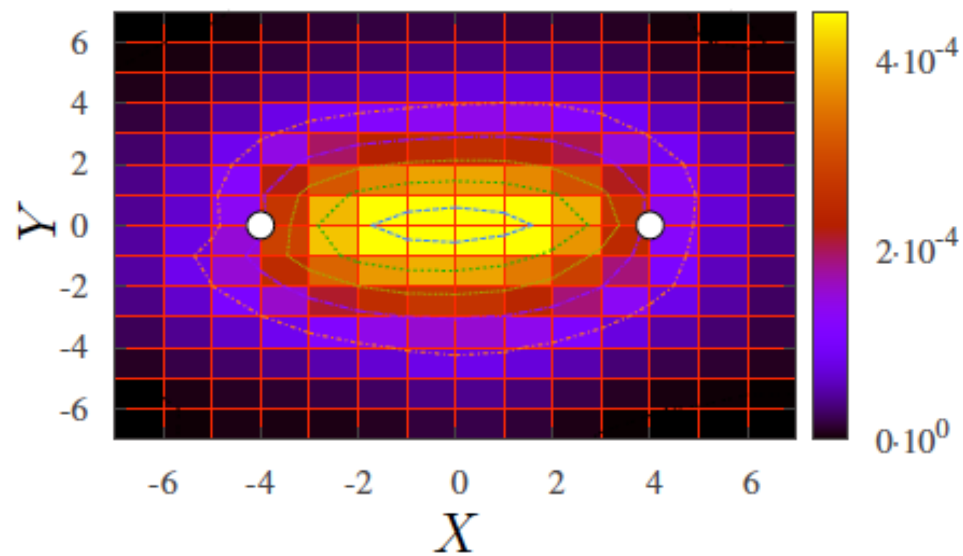
$$\langle P|\bar{q}q|P\rangle = \langle P|\int d^3x\ \bar{q}(x)q(x)|P\rangle - \langle 0|\bar{q}q|0\rangle V_P$$

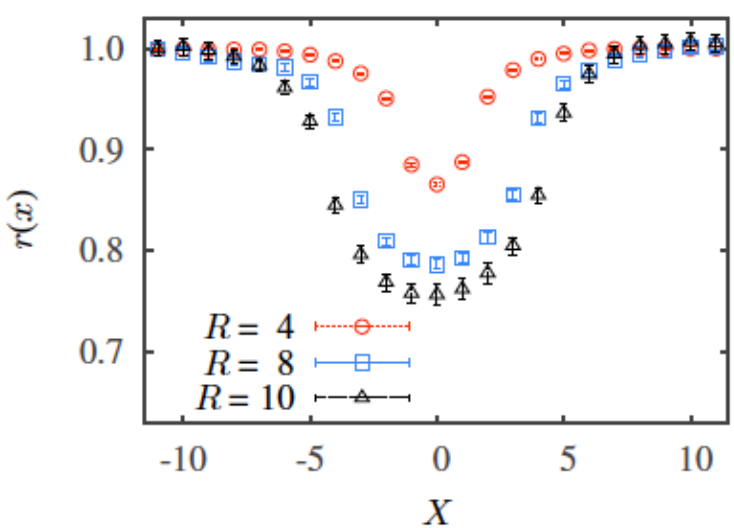
This difference results from the partial **restoration** of spontaneously broken chiral symmetry inside the hadron

e.g., Donoghue, Nappi '86

Partial restoration of chiral symmetry inside the nucleon



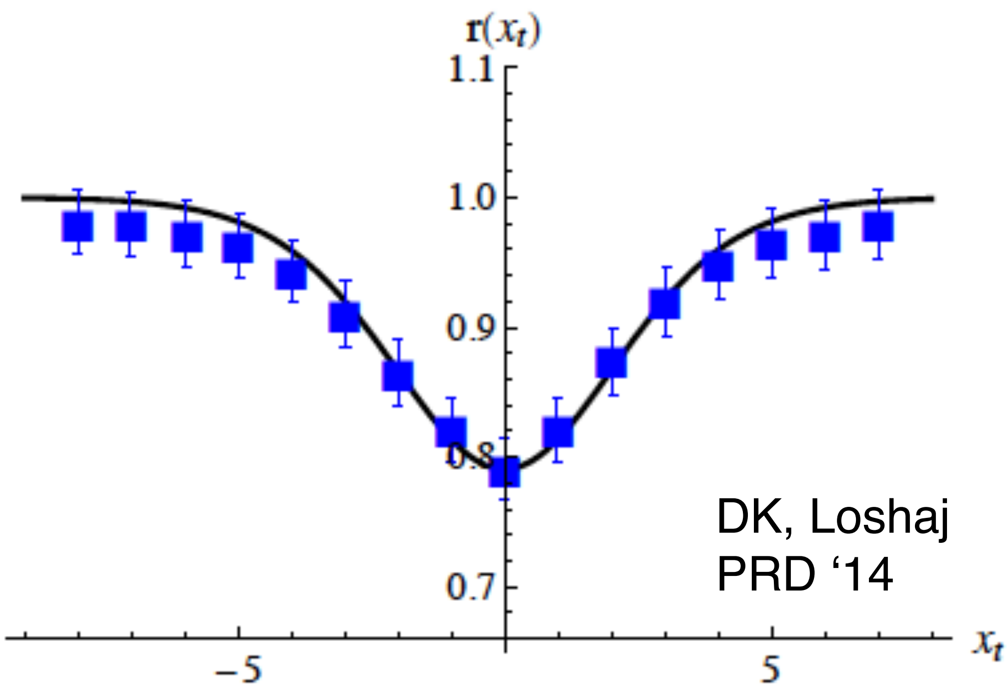




Significant suppression of the local quark condensate by the confining flux tube!

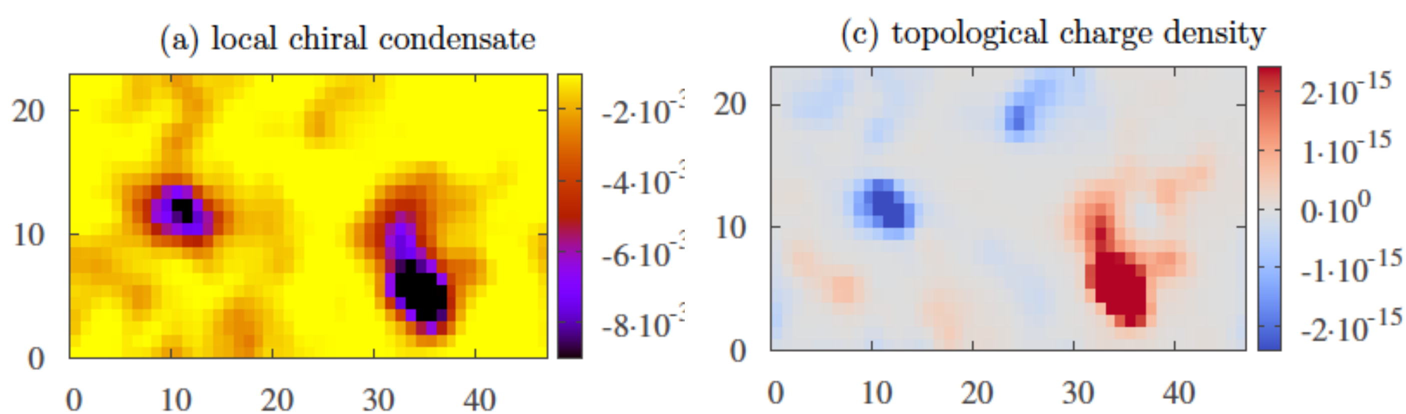
Iritani, Cossu, Hashimoto
arXiv:1502.04845 PRD

Partial restoration of chiral symmetry inside the nucleon



A possible mechanism of chiral condensate suppression involves the chiral anomaly –

so the entire mass of the proton might originate from anomalies – scale and chiral !

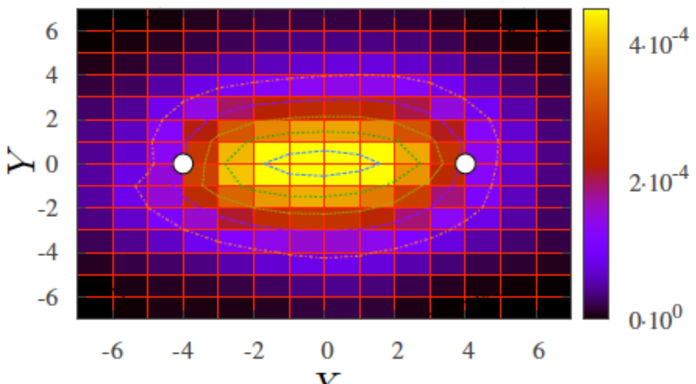


The proton mass as a result of the vacuum polarization induced by the presence of the proton

$$\Theta_{\alpha}^{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^{a} + \sum_{l=u,d,s} m_l \bar{q}_l q_l ,$$

Polarization of the gluon field;

~ 90% of the proton's mass ?



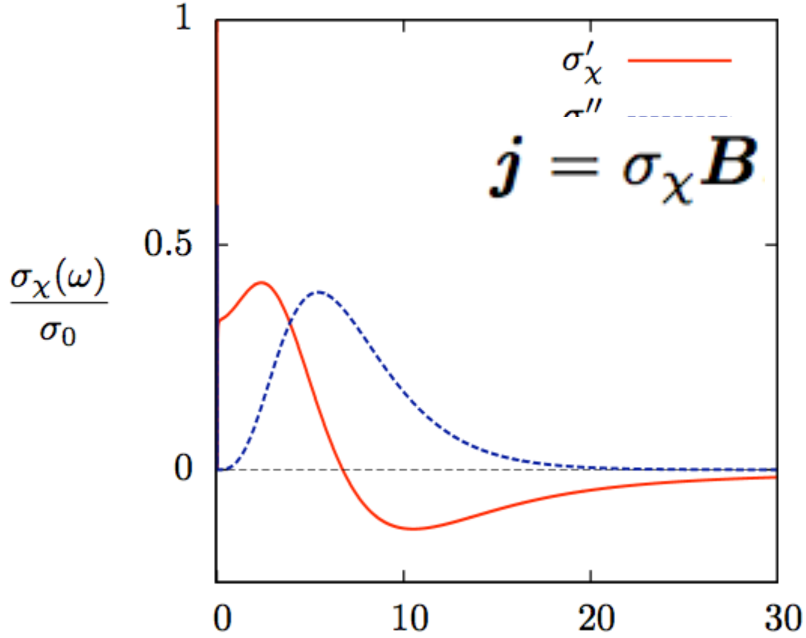
Polarization of the quark condensate;

numerically, ~ 80 MeV using

Y.-B.Yang et al
arXiv:1511.09089

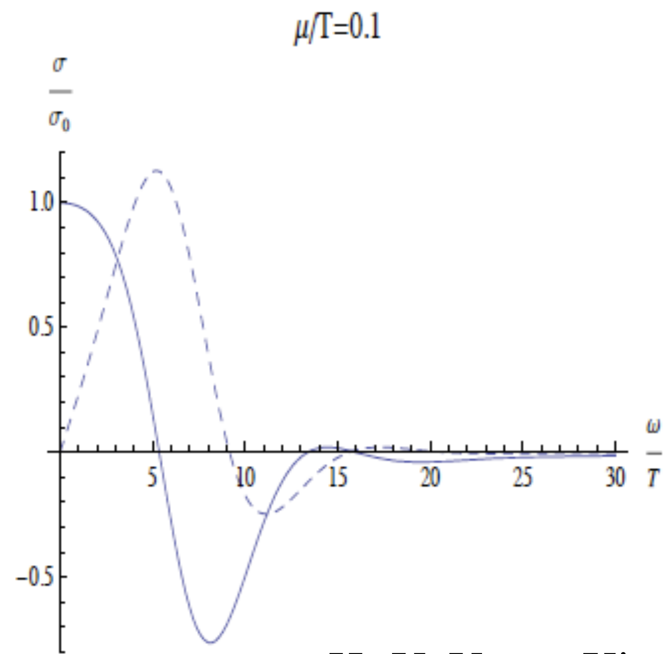
Dynamical chiral magnetic effect

Weak coupling



D.K., H. Warringa ω/T
Phys Rev D80 (2009) 034028

Strong coupling



H.-U. Yee, arXiv:0908.4189,
JHEP 0911:085, 2009;

DK,
M.Stephanov,
H.-U.Yee.
1612.01674
PRD'17

A.Rebhan, A.Schmitt, S.Stricker JHEP 0905, 084 (2009), G.Lifshytz, M.Lippert, arXiv:0904.4772;.A. Gorsky, P. Kopnin, A. Zayakin, arXiv:1003.2293, A.Gynther, K. Landsteiner, F. Pena Benitez, JHEP 1102 (2011) 110; V. Rubakov, arXiv:1005.1888, C. Hoyos, T. Nishioka, A. O'Bannon, JHEP1110 (2011) 084; …

CME persists at strong coupling - **hydrodynamical formulation?**

The CME in relativistic hydrodynamics: The Chiral Magnetic Wave

DK, H.-U. Yee, arXiv:1012.6026 [hep-th]; PRD

$$\vec{j}_V = \frac{N_c\, e}{2\pi^2}\mu_A \vec{B}; \quad \vec{j}_A = \frac{N_c\, e}{2\pi^2}\mu_V \vec{B},$$

CME   Chiral separation

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c\, e\vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}$$

Propagating chiral wave: (if chiral symmetry is restored)

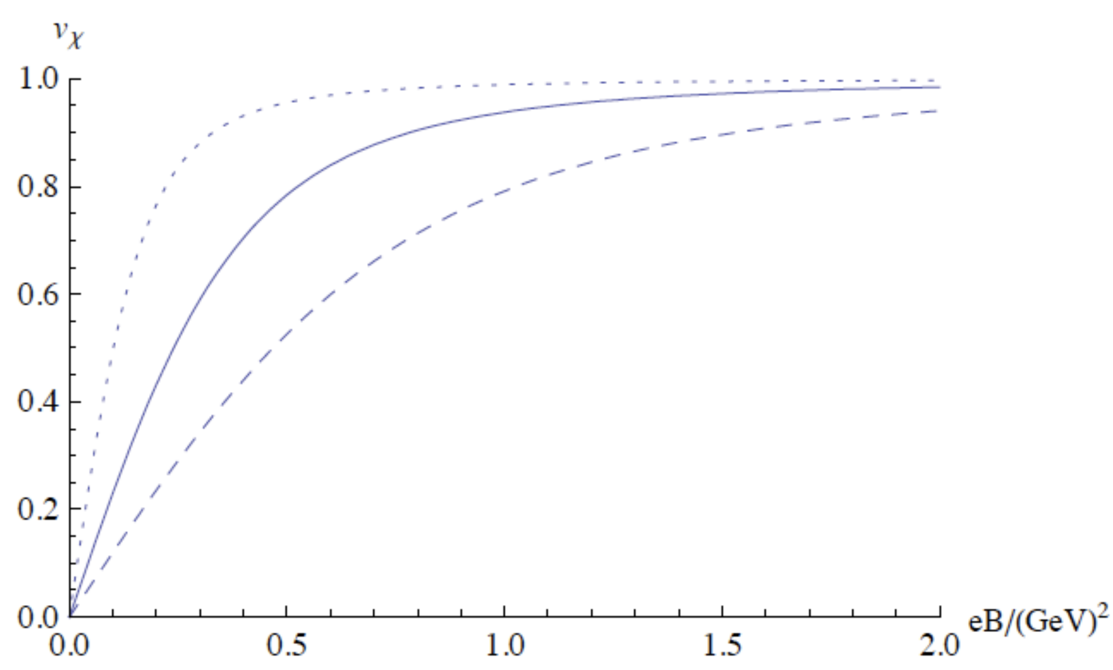
$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2}\partial_1 - D_L \partial_1^2\right) j^0_{L,R} = 0$$

Gapless collective mode is the carrier of CME current in MHD:

$$\omega = \mp v_\chi k - i D_L k^2 + \cdots$$

The Chiral Magnetic Wave:
oscillations of electric and chiral charges coupled by the chiral anomaly

In strong magnetic field, CMW propagates with the speed of light!



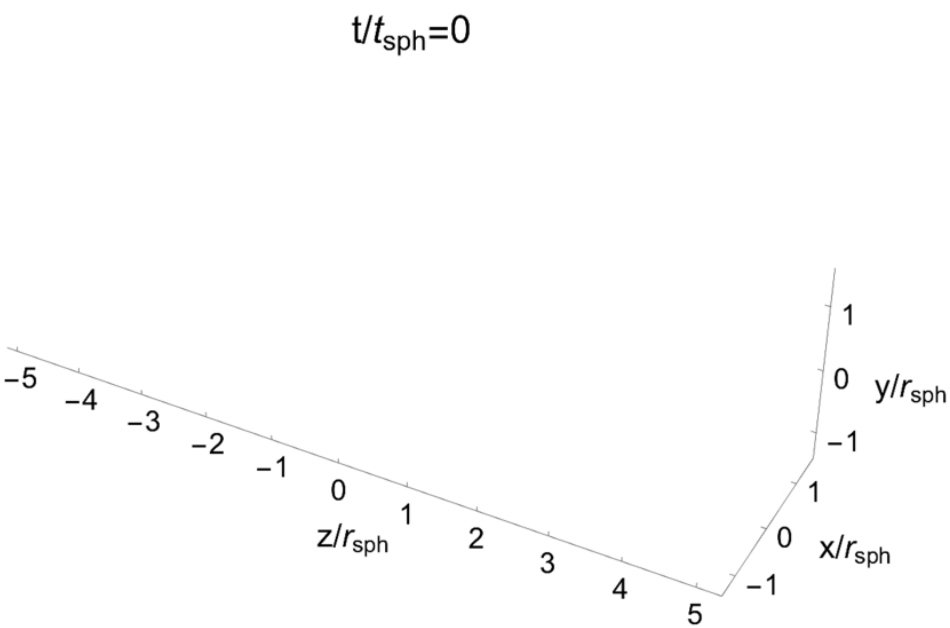
DK, H.-U. Yee, Phys Rev D'11

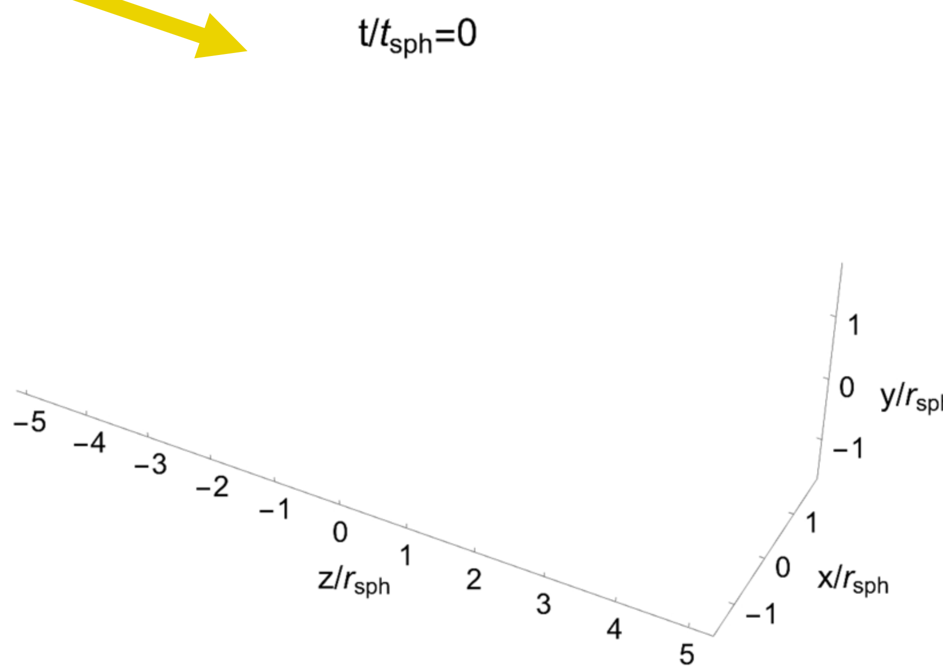
M. Mace, N. Mueller, S. Schlichting, S. Sharma, arxiv:1704.05887; PRD'17

Chiral Magnetic Wave in real time!

Anomalous transport in real time

j_a^0 :axial charge $\vec{B}$ j_v^0 :vector charge





Static U(1) magnetic field in z-dir