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Chiral Matter

from quarks to quantum materials

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Contents

1. Chirality in gauge theories

2. Chiral magnetic effect (CME) and anomaly-induced transport

3. CME in heavy ion collisions

4. CME in condensed matter

5. Chirality, quantum entanglement and the parton model

Chirality: the definition

Greek word: χειρ (cheir) - hand

Lord Kelvin (1893):

"I call any geometrical figure, or groups of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself."

Light and electromagnetism

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

THE
LONDON, EDINBURGH AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FOURTH SERIES.]

MARCH 1861.

XXV. *On Physical Lines of Force.* By J. C. MAXWELL, Professor of Natural Philosophy in King's College, London*.

PART I.—*The Theory of Molecular Vortices applied to Magnetic Phenomena.*

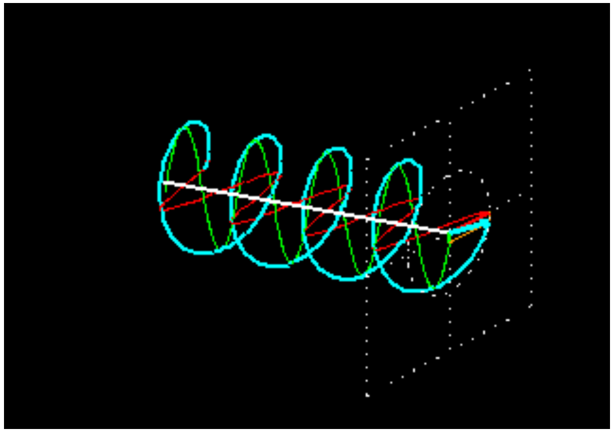
IN all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the *magnitude* and *direction* of the force which would act on a given body, if placed in a given position.

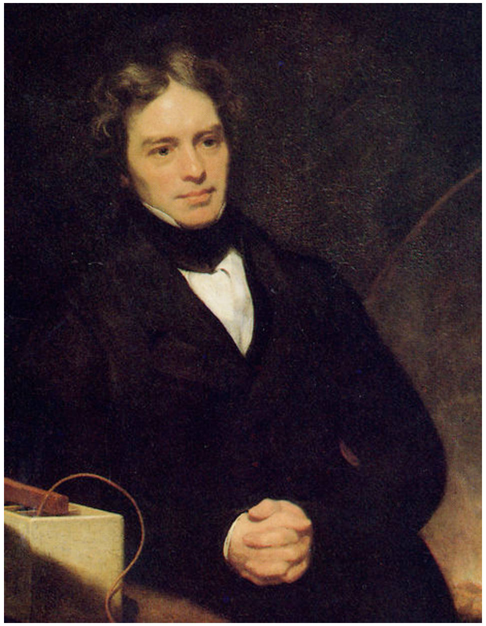
In the case of a body acted on by the gravitation of a sphere, this force is inversely as the square of the distance, and in a straight line to the centre of the sphere. In the case of two attracting spheres, or of a body not spherical, the magnitude and direction of the force vary according to more complicated laws. In electric and magnetic phenomena, the magnitude and direction of the resultant force at any point is the main subject of investigation. Suppose that the direction of the force at any point is known, then, if we draw a line so that in every part of its course it coincides in direction with the force at that point, this line may be called a *line of force*, since it indicates the direction of the force in every part of its course.

By drawing a sufficient number of lines of force, we may indicate the direction of the force in every part of the space in which it acts.

Thus if we strew iron filings on paper near a magnet, each filing will be magnetized by induction, and the consecutive filings will unite by their opposite poles, so as to form fibres, and these fibres will *indicate* the direction of the lines of force. The beautiful illustration of the presence of magnetic force afforded by this experiment, naturally tends to make us think of

James C. Maxwell, 1831-1879

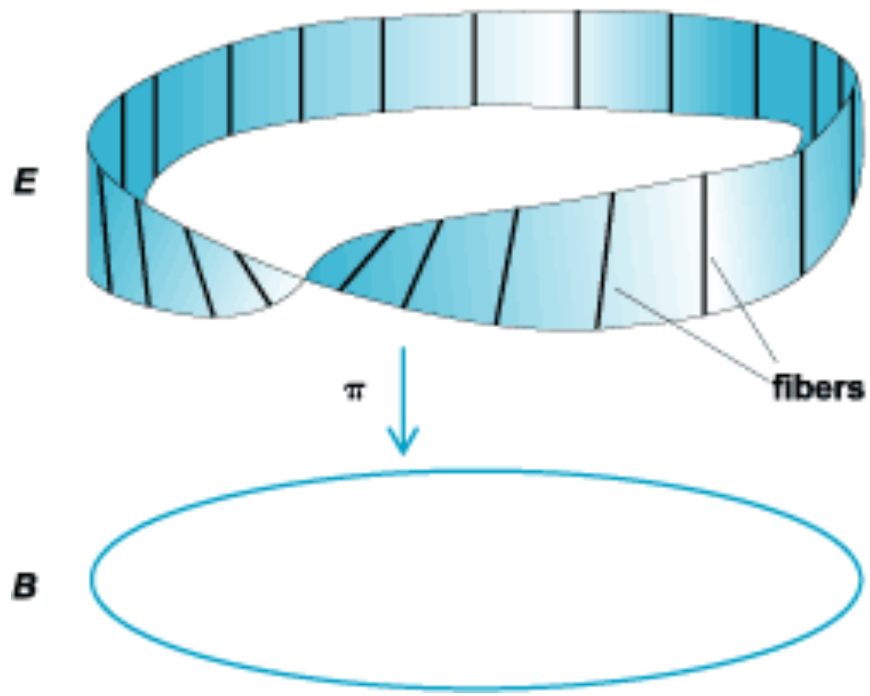




Maxwell theory is left-right symmetric

Michael Faraday, 1791-1867

Gauge fields and topology



NB: Maxwell electrodynamics as a curvature of a line bundle

Möbius strip, the simplest nontrivial example of a fiber bundle

Gauge theories "live" in a fiber bundle space that possesses non-trivial topology (knots, links, twists,...)

Chern-Simons forms

6. Applications to 3-manifolds

In this section M will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection θ and curvature tensor Ω. For A, B skew symmetric matrices, the specific formula for P_1 shows $P_1(A \otimes B) = -(1/8\pi^2)\operatorname{tr} AB$. Calculating from (3.5) shows

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\}\,. \tag{6.1}$$

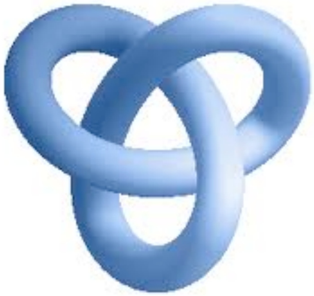
What does it mean for a gauge theory?

Chern-Simons theory

CHARACTERISTIC FORMS

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} . \tag{6.1}$$

What does it mean for electromagnetism?



Geometry	⟷	Physics
Riemannian connection	⟷	Gauge field
Curvature tensor	⟷	Field strength tensor

$$\boxed{S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)}$$

“magnetic helicity”

Chern-Simons form and circularly polarized light

How to describe the helicity of the circularly polarized light?

Magnetic helicity itself does not obey electric-magnetic symmetry of Maxwell equations in vacuum:

$$\mathbf{E} \to \cos\theta\ \mathbf{E} + \sin\theta\ \mathbf{B}$$

$$\mathbf{B} \to \cos\theta\ \mathbf{B} - \sin\theta\ \mathbf{E}$$

Heaviside, 1892
Larmor, 1897

We can however enforce this symmetry by introducing, in addition to the magnetic helicity, the dual pseudovector gauge potential C. In Coulomb gauge C is defined by:

Bateman, 1915

$$\nabla\mathbf{A} = \nabla\mathbf{C} = 0$$

$$\mathbf{E} = -\nabla \times \mathbf{C} = -\dot{\mathbf{A}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\dot{\mathbf{C}}$$

Optical helicity
of the circularly polarized light

Electric-magnetic transformation

$$\mathbf{E} \to \cos\theta\ \mathbf{E} + \sin\theta\ \mathbf{B}$$
$$\mathbf{B} \to \cos\theta\ \mathbf{B} - \sin\theta\ \mathbf{E}$$

is induced by

$$\mathbf{A} \to \cos\theta\ \mathbf{A} + \sin\theta\ \mathbf{C}$$
$$\mathbf{C} \to \cos\theta\ \mathbf{C} - \sin\theta\ \mathbf{A}$$

We can now define the **optical helicity** by adding CS terms for A and C:

$$H \equiv \frac{1}{2}\int d^3x\,(\mathbf{A}\cdot(\nabla\times\mathbf{A}) + \mathbf{C}\cdot(\nabla\times\mathbf{C})) = \frac{1}{2}\int d^3x\,(\mathbf{A}\cdot\mathbf{B} - \mathbf{C}\cdot\mathbf{E})$$

Candlin, 1965; Trueba,Ranada, 1996; Afanasiev,Stepanovsky, 1996; Cameron, Barnett, Yao, 2012

Optical helicity of the circularly polarized light

The optical helicity

$$H = \frac{1}{2} \int d^3x \, (\mathbf{A} \cdot \mathbf{B} - \mathbf{C} \cdot \mathbf{E})$$

is invariant under electric-magnetic symmetry

$$\mathbf{A} \to \cos\theta \ \mathbf{A} + \sin\theta \ \mathbf{C}$$

$$\mathbf{C} \to \cos\theta \ \mathbf{C} - \sin\theta \ \mathbf{A}$$

It is a T-even, P-odd quantity that is conserved *in the absence of interactions with chiral (P-odd) matter*:

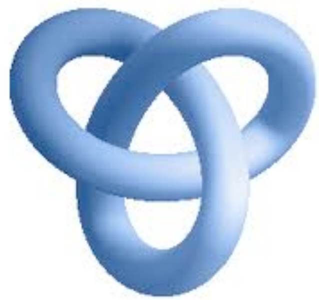
$$\frac{dH}{dt} = 0$$

Chern-Simons theory

CHARACTERISTIC FORMS

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12}\wedge\theta_{13}\wedge\theta_{23} + \theta_{12}\wedge\Omega_{12} + \theta_{13}\wedge\Omega_{13} + \theta_{23}\wedge\Omega_{23}\}\,. \tag{6.1}$$

What does it mean for electromagnetism?



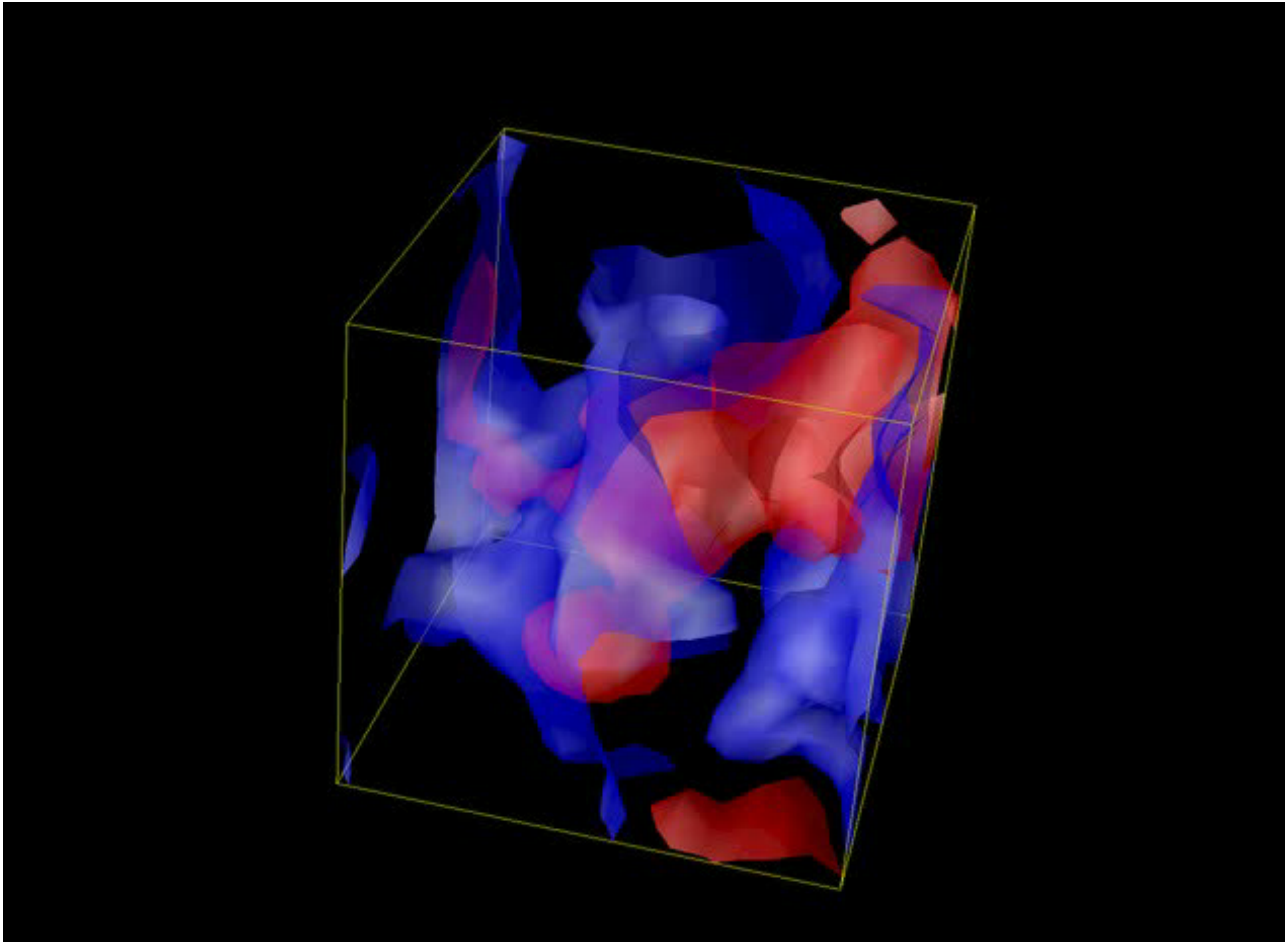
Geometry	$\longleftrightarrow$	Physics
Riemannian connection	$\longleftrightarrow$	Gauge field
Curvature tensor	$\longleftrightarrow$	Field strength tensor

$$\boxed{S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)}$$

"magnetic helicity" Non-Abelian helicity

“Topological foam” in QCD vacuum, (3+1) Dimensions

ITEP Lattice Group

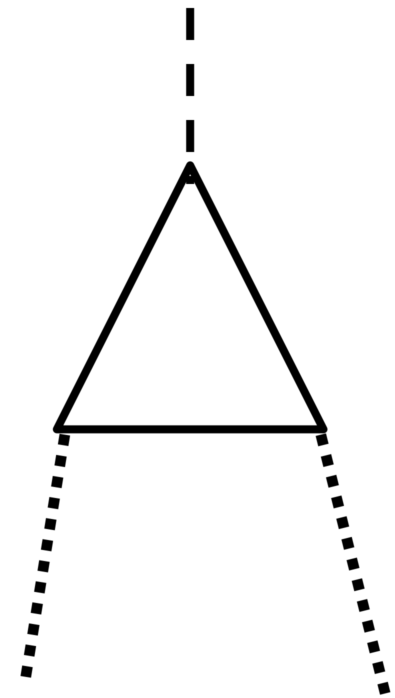


Chirality in electrodynamics, (3+1)D: Maxwell-Chern-Simons theory

$$\mathcal{L}_{\mathrm{MCS}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\, P_\mu J^\mu_{CS}$$

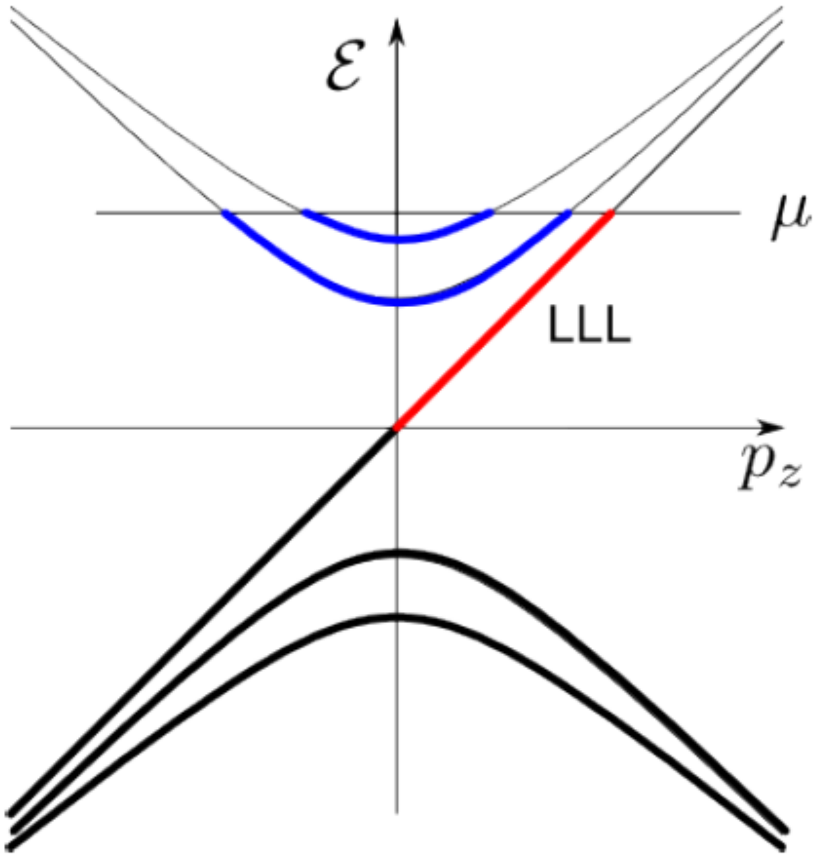
$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu \theta = (\dot{\theta}, \vec{P})$$

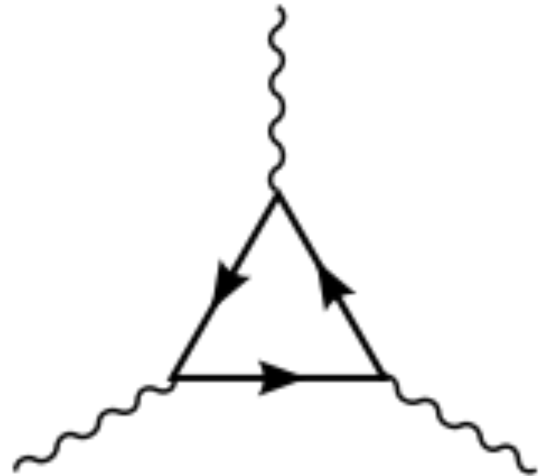
$$\boxed{\begin{gathered} \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + \underline{c\left(\dot{\theta}\vec{B} - \vec{P} \times \vec{E}\right)}, \\ \vec{\nabla} \cdot \vec{E} = \rho + \underline{c\vec{P} \cdot \vec{B}}, \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \\ \vec{\nabla} \cdot \vec{B} = 0, \end{gathered}}$$



DK, Ann.Phys. 325 (2010) 205

Chiral anomaly





In classical background fields (E and B), chiral anomaly induces a collective motion in the Dirac sea

Adler; Bell, Jackiw; Nielsen, Ninomiya; …

Problem:

Derive the action describing a source of chirality descried by θ(x,t) for non-linear electrodynamics, e.g. for Born-Infeld action:

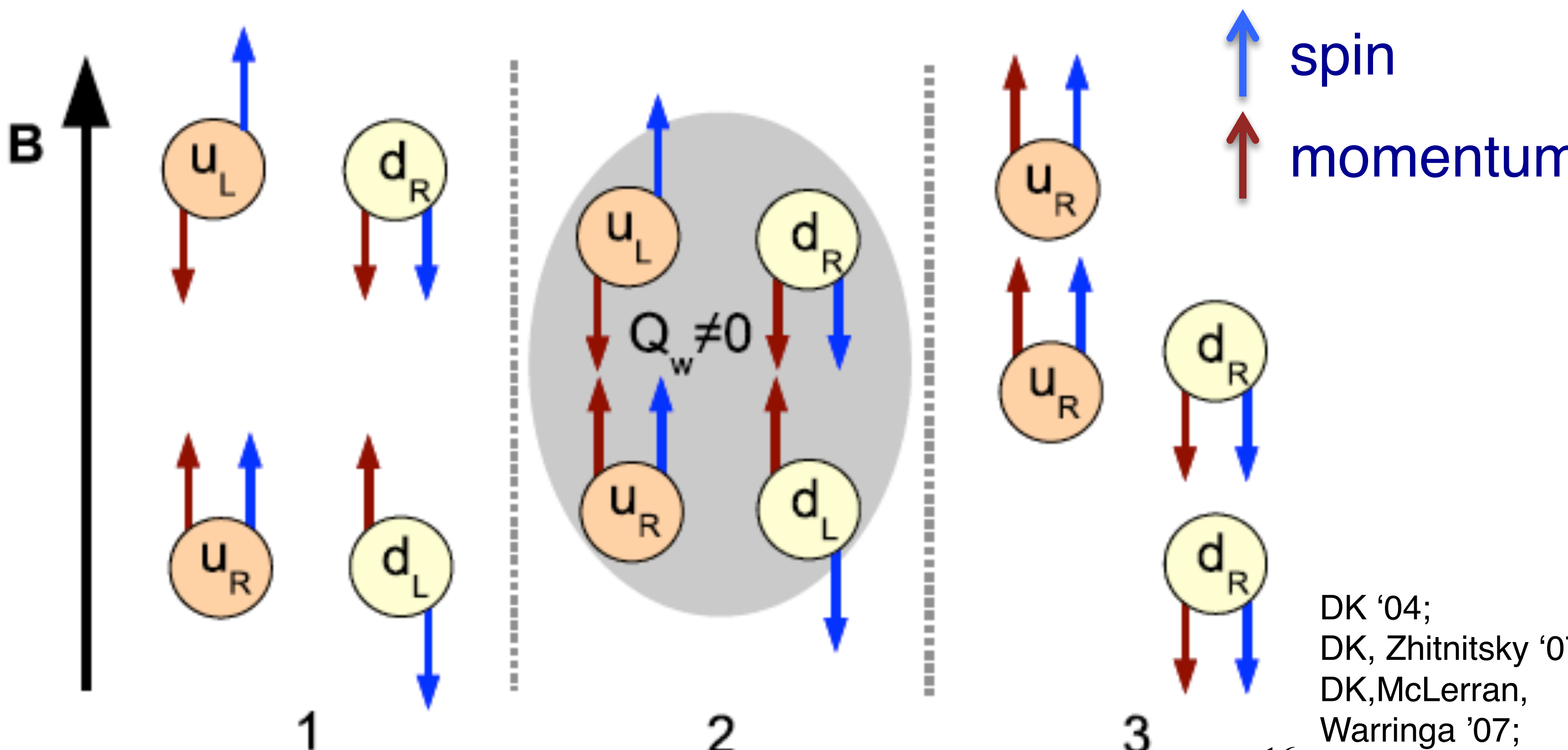
$$\mathcal{L} = -b^2 \sqrt{1 - \frac{E^2 - B^2}{b^2} - \frac{(\mathbf{E} \cdot \mathbf{B})^2}{b^4}} + b^2$$

Note: the BI action obeys the electric-magnetic symmetry

Applications include the generation of second harmonic in chiral nanotubes, see F.Qin, “Superconductivity in a chiral nanotube” Nature Comm. 2017; WS_2

Chirality in 3D:
the Chiral Magnetic Effect

$\boxed{\text{chirality + magnetic field = current}}$



Review: DK, arxiv:1312.3348 (Prog.Part.Nucl.Phys'14)

DK '04;
DK, Zhitnitsky '07
DK,McLerran, Warringa '07;
Fukushima, DK, Warringa '08

Early work on currents in magnetic field due to P violation

(see DK, Prog.Part.Nucl.Phys. 75 (2014) 133 for a complete (?) list of references)

A.Vilenkin (1980) “Equilibrium parity-violating current in a magnetic field”;
(1980) “Cancellation of equilibrium parity-violating currents”

G. Eliashberg (1983) JETP 38, 188
L. Levitov, Yu.Nazarov, G. Eliashberg (1985) JETP 88, 229

M. Joyce and M. Shaposhnikov (1997) PRL 79, 1193;
M. Giovannini and M. Shaposhnikov (1998) PRL 80, 22

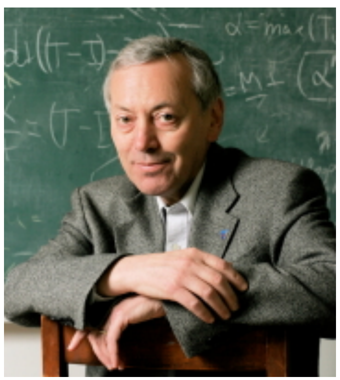
A. Alekseev, V. Cheianov, J. Frohlich (1998) PRL 81, 3503

PHYSICAL REVIEW D VOLUME 22, NUMBER 12 15 DECEMBER 1980

Equilibrium parity-violating current in a magnetic field

Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155
(Received 1 August 1980)

It is argued that if the Hamiltonian of a system of charged fermions does not conserve parity, then an equilibrium electric current parallel to $\vec{B}$ can develop in such a system in an external magnetic field $\vec{B}$. The equilibrium current is calculated (i) for noninteracting left-handed massless fermions and (ii) for a system of massive particles with a Fermi-type parity-violating interaction. In the first case a nonzero current is found, while in the second case the current vanishes in the lowest order of perturbation theory. The physical reason for the cancellation of the current in the second case is not clear and one cannot rule out the possibility that a nonzero current appears in other models.



But: no current in <u>equilibrium</u>

C.N. Yang

PHYSICAL REVIEW D VOLUME 22, NUMBER 12 15 DECEMBER 1980

Cancellation of equilibrium parity-violating currents

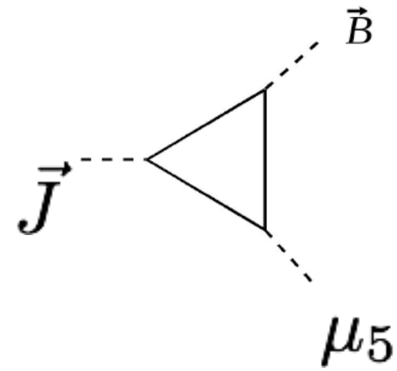
Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155

Chiral Magnetic Effect

DK'04; K.Fukushima, DK, H.Warringa, PRD'08;
Review and list of refs: DK, arXiv:1312.3348

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of B, vector e.m. current is generated:



$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2}\left(F_L^{\mu\nu}\tilde{F}_{L,\mu\nu} - F_R^{\mu\nu}\tilde{F}_{R,\mu\nu}\right)$$

Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

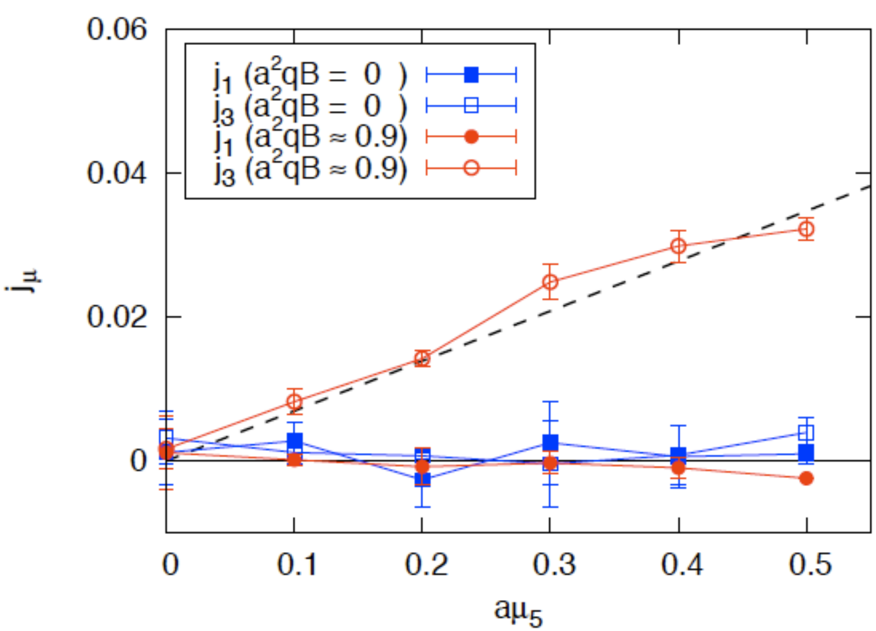
$$\boxed{\vec{J} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}}$$

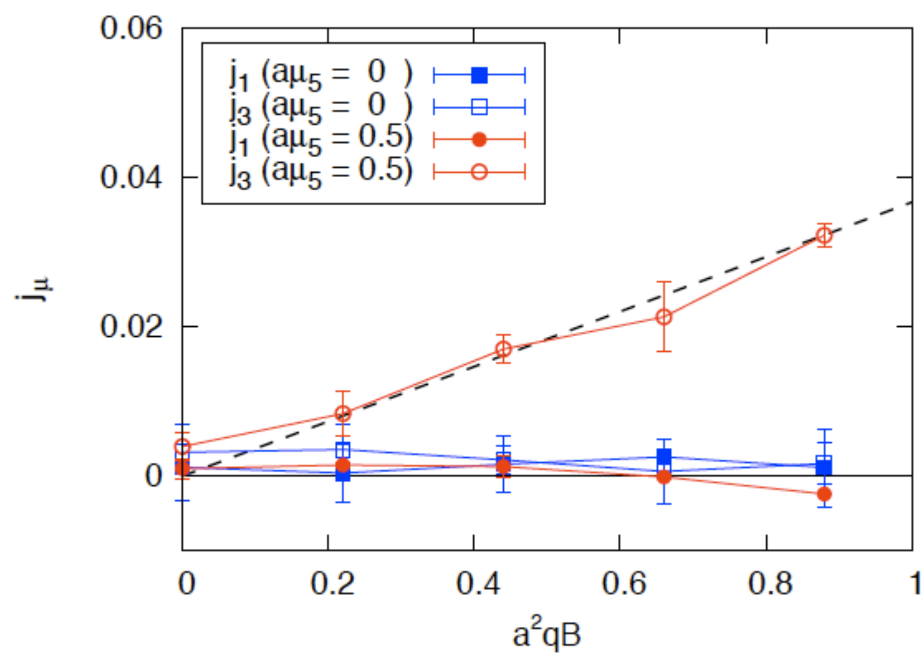
Coefficient is fixed by the axial anomaly, no corrections arXiv:1105.0385, PRL

Chiral magnetic effect in lattice QCD with chiral chemical potential

Arata Yamamoto
Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan
(Dated: May 3, 2011)

We perform a first lattice QCD simulation including two-flavor dynamical fermion with chiral chemical potential. Because the chiral chemical potential gives rise to no sign problem, we can exactly analyze a chirally asymmetric QCD matter by the Monte Carlo simulation. By applying an external magnetic field to this system, we obtain a finite induced current along the magnetic field, which corresponds to the chiral magnetic effect. The obtained induced current is proportional to the magnetic field and to the chiral chemical potential, which is consistent with an analytical prediction.





Systematics of anomalous conductivities

	Magnetic field	Vorticity
Vector current	$\dfrac{\mu_A}{2\pi^2}$	$\dfrac{\mu\mu_A}{2\pi^2}$
Axial current	$\dfrac{\mu}{2\pi^2}$	$\dfrac{\mu^2+\mu_A^2}{4\pi^2}+\dfrac{T^2}{12}$

Chiral magnetic conductivity: discrete symmetries

P – parity
T – time reversal

$$\vec{J} = \boxed{\frac{e^2}{2\pi^2}\,\mu_5}\,\vec{B}$$

P-odd
T-odd

P-even
T-odd

P-odd

<u>P-odd effect!</u>

Effect persists in hydrodynamics!

T-even

<u>Non-dissipative current!</u>

(topologically protected)

cf Ohmic conductivity:

$$\vec{J} = \sigma \vec{E}$$

T-odd, dissipative

CME as a new type of superconductivity

London theory of superconductors, '35:

$$\vec{\mathbf{J}} = -\lambda^{-2}\vec{\mathbf{A}} \qquad \nabla \cdot \vec{\mathbf{A}} = 0$$

Fritz and Heinz London

$$\vec{\mathbf{E}} = -\dot{\vec{\mathbf{A}}} \qquad \boxed{\vec{\mathbf{E}} = \lambda^2 \dot{\vec{\mathbf{J}}}}$$

assume that chirality is conserved:

$$\mu_5 \sim \vec{E}\vec{B}\ t$$

CME:

$$\vec{J} \sim \mu_5\ \vec{B}$$

for $\vec{E}||\vec{B}$

$$\boxed{\vec{E} \sim B^{-2}\,\dot{\vec{J}}}$$

superconducting current, tunable by magnetic field!

DK, arXiv:1612.05677